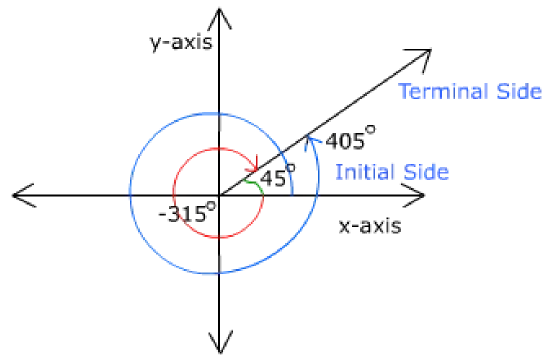
TASK: Angles play a critical role in creating modern architecture. They are also fundamental in trigonometry. In this lesson, we begin our study of trigonometry by looking at angles and methods for measuring them. We will review a lot of what we learned in Algebra 2 and begin to expand on this knowledge.

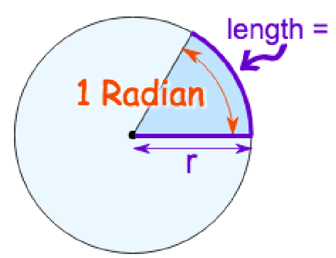
Part I: Drawing angles & finding coterminal angles

- When working with angles, we first must be able to properly draw the diagrams and be familiar with the vocabulary.
- We always draw the angles in ***standard position***, with the ***initial side*** beginning on the positive side of the x-axis.
- The example to the right shows three ***coterminal angles*** (they all share the same terminal side)



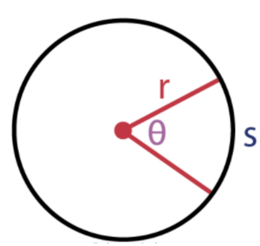
Part II: The length of a circular arc

- In Algebra 2, we learned that 1 revolution (or 360°) is 2π radians, thus π radians = 180°
- One radian is the measure of the central angle of a circle that intercepts an arc equal in length to the radius of the circle.
- To convert from degrees to radians, we can multiply by $\frac{\pi}{180}$.
  - Here's a video if you want a refresher.
  - Below is also an example demonstrating the conversion.



$$15^\circ \to 15^\circ\left(\frac{\pi}{180^\circ}\right) \to \overset{1}{\cancel{15}}{}^\circ\left(\frac{\pi}{\underset{12}{\cancel{180^\circ}}}\right) \to \frac{\pi}{12}$$

- We can also find use radians when working with central angles and arc length (think back to Geometry!)
  - We can use the formula $s = r\theta$ where:
    - s = arc length
    - r = radius
    - θ = central angle (measured in radians)



Example: A circle with a radius of 6 inches has an intercepted arc that measures 15 inches. What is the central angle measurement?

If we use the formula $s = r\theta$, we can substitute in 6 inches and 15 inches to then solve for θ.
$15 = 6\theta$ → after dividing both sides by 6, we find $2.5\ radians = \theta$

Part III: Linear and Angular Speed

If a point is in motion on a circle of radius ***r*** through an angle of θ radians in time ***t***, then its linear speed is $v = \frac{s}{t}$, where ***s*** is the arc length, and it's angular speed is $\omega = \frac{\theta}{t}$.

Example: The hard drive in a computer rotates at 3600 revolutions per minute. This angular speed, expressed in revolutions per minute, can also be expressed in revolutions per second, radians per minute, and radians per second. Using 2π radians = 1 revolution, we express the angular speed of a hard drive in radians per minute as follows.

$$\frac{\theta}{t} = \frac{3600\ revolutions}{1\ minute} \bullet \frac{2\pi\ radians}{1\ revolution} = \frac{7200\pi\ radians}{1\ minute}$$

APPLICATION/PRACTICE:

1. Draw and label each angle in standard position:
   (a) $\theta = \frac{\pi}{4}$ (b) $\alpha = -\frac{3\pi}{4}$ (c) $\beta = \frac{9\pi}{4}$

2. Find a positive angle less than 360° that is coterminal with each of the following:
   (a) 400° (b) -135° (c) $\frac{17\pi}{6}$ (Challenge: Leave in radian form & include π in your answer. Compare to an entire revolution)

3. A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 120°.

4. A circle has a radius of 10 inches. Find the length of the arc intercepted by a central angle of 120°.

5. Find the positive radian measure of the angle that the second hand of a clock moves through at 55 seconds.

6. Assuming Earth to be a sphere of radius 400 miles, how many miles north of the Equator is Miami, Florida, if it is 26° north from the Equator? Round your answer to the nearest mile.