<u>NEW OUTCOME:</u> Analyze and make predictions about arithmetic and geometric sequences and series.

CLICK HERE TO FOLLOW ALONG WITH A RECORDED READING OF THESE NOTES

PART I: When you do things in a sequence, you do them in order: first, second, third, and so on. A ***sequence*** is an ordered list of numbers, each called a ***term*** of the sequence. Because a sequence maps a set of ordered integers to the terms of the sequence, a sequence is a function whose *domain* is the subset of integers and *range* is the set of values of the terms.

A sequence is often described by an equation, or ***rule***. The domain for the rule often begins with the <u>position</u> number n = 1, referring to the first number in the list. Consider the sequence *a(n)* describing the cubes of the first 5 positive integers.

$a(n) = n^3$ Domain: {1, 2, 3, 4, 5} Range: {1, 8, 27, 64, 125}

We can describe the terms in the function as function notation or subscript notation.

$a(1) = 1$ $a_1 = 1$ The first term is 1.	$a(2) = 8$ $a_2 = 8$ The second term is 8.	$a(3) = 27$ $a_3 = 27$ The third term is 27.	$a(4) = 64$ $a_4 = 64$ The fourth term is 64.	$a(5) = 125$ $a_5 = 125$ The fifth term is 125.

APPLICATION/PRACTICE OF NEW SKILL:

1. The front row of a movie theater has 11 seats, each row after that has an additional 3 seats. The number of seats in the *n*th row, where $n \geq 1$, can be described by a sequence with the rule shown using both function and subscript notation.

$$a(n) = 11 + 3(n - 1) \text{ or } a_n = 11 + 3(n - 1)$$

How many seats are in the 8th row? We can also notate this as $a(8)$ or a_8.

2. Write the first six terms of the sequence $a(n) = 132 - 11n$, beginning with $n = 1$.

3. Write the first six terms of the sequence $a_n = (n + 3)^2$, beginning with $n = 1$.

4. For a regular polygon with n sides, the rule $a_n = \frac{180(n-2)}{n}$ gives the measure of an interior angle.

   (a) In this context, what are the domain and range of the sequence that this rule describes?

   (b) Write the first 10 terms of the sequence.

PART II: There are many times that it is useful to know the sum of the terms in a sequence. A ***series*** is the expression formed by adding the terms of the sequence. For example, 5 + 10 + 15 + 20 + 25 is the series formed by adding the terms of the sequence {5, 10, 15, 20, 25}.

To represent a series efficiently, you can use ***sigma notation***, named for the Greek letter for S, which is represented by the symbol $\sum$. For example, the terms of the sequence above can be represented by the rule $5n$, where $1 \le n \le 5$. Sigma notation uses the index of summation using the letter k or another letter, to count the terms from the lower ***limit of summation*** (in this case 1) to the ***upper limit of summation*** (in this case 5).

$$\sum_{k=1}^{5} 5k$$

←upper limit of summation

← sequence rule using index k

← lower limit of summation

So, $\sum_{k=1}^{5} 5k = 5(1) + 5(2) + 5(3) + 5(4) + 5(5) = 5 + 10 + 15 + 20 + 25$

Sometimes the summations can be very long and annoying to add up such as $\sum_{k=1}^{30} k + 2$, we would have to add up all terms from 1 to 30. In this case we have a formula to help shorten the time we spend summing, $S_n = \frac{n(a_1 + a_n)}{2}$. This formula allows us to use the first term a_1, last term a_n, and number of terms n to find the summation, or series. For example, we can quickly find the first term where k = 1, 1 + 2 = 3, the last term, where k = 30, 30 + 2 = 32 and enter this information in the formula $\sum_{k=1}^{30} k + 2 = \frac{30(3+32)}{2} = 525$.

APPLICATION/PRACTICE OF NEW SKILL:

1. Write out the series in function notation and find the sum for $\sum_{k=1}^{10} 2k - 10$.

2. Use the summation formula to find the sum for $\sum_{k=1}^{16} k - 8$.