AIM: How can we add and subtract rational expressions?

We already know how to add and subtract ratios by creating equivalent fractions with a common denominator. Without a common denominator, the ratios cannot be combined. In the example below the left fraction was multiplied by $\frac{2}{2}$ and the right fraction was multiplied by $\frac{3}{3}$ to create equivalent fractions that now have a common denominator of 6.

the original fractions: $\frac{1}{3}+\frac{1}{2}$

with a common denominator: $\frac{2}{6}+\frac{3}{6}$

result: $\frac{5}{6}$

When working with polynomials, we still want common denominators. However, making common denominators requires more complex arithmetic when we are now including variables. Remember, we multiply the numerator and denominator by the amount needed (this allows us to create an equivalent fraction). In the example below the common denominator is $6x^2$ so the left fraction is multiplied by $\frac{3}{3}$ and the right fraction is multiplied by $\frac{2x}{2x}$.

Add: $\frac{10}{2x^2}+\frac{5}{3x}$

$\frac{10\cdot 3}{2x^2\cdot 3}+\frac{5\cdot 2x}{3x\cdot 2x}$

$\frac{30}{6x^2}+\frac{10x}{6x^2}=\frac{30+10x}{6x^2}$

$\frac{10(3+x)}{6x^2}=\frac{5(3+x)}{3x^2}$ *or* $\frac{15+5x}{3x^2}$

When working with higher degree polynomials we can do the same thing, but the factors are not as obvious to see and often require us to ***factor the denominators first***. In the example below each denominator first needed to be factored in order to see what was needed to make a common denominator. Read through the example and copy into your notes.

$$\frac{5x-1}{x^2-3x+2}+\frac{3}{2x-4}=\frac{5x-1}{(x-1)(x-2)}+\frac{3}{2(x-2)}=$$

$$=\frac{2(5x-1)}{2(x-1)(x-2)}+\frac{3(x-1)}{2(x-1)(x-2)}=$$

$$=\frac{2(5x-1)+3(x-1)}{2(x-1)(x-2)}=$$

$$=\frac{13x-5}{2(x-1)(x-2)}$$

EXPLORE/PRACTICE: Simplify each expression below.

(1) $\frac{3x^2-2x+1}{x-2}-\frac{2x^2+3x-5}{x-2}$

(2) $\frac{x^2+3x-2}{(x+5)(x-2)}+\frac{4x+12}{(x+5)(x-2)}$

(3) $\frac{4x-1}{5x}+\frac{x+5}{10}$

(4) $\frac{4}{5y-15}+\frac{5}{y^2-9}$

(5) $\frac{x}{x^2-5x+6}-\frac{3}{x-3}$

(6) $\frac{2}{x^2-4}-\frac{1}{x^2+2x}$

(7) $\frac{x-3}{x^2-9x+20}+\frac{2}{x^2-6x+8}$

(8) $\frac{x}{x^2+9x+20}-\frac{4}{x^2+7x+12}$