

Name ANSWER KEY

AIM: Practice combining and simplifying radicals

LEVEL ONE:

I. Rewrite in exponential form then simplify.

(a) $\sqrt[3]{-64} = (-4^3)^{\frac{1}{3}} = -4$
 (b) $\sqrt[4]{16x^8} = (2^4 x^8)^{\frac{1}{4}} = 2x^2$
 (c) $\sqrt{100x^6y^4} = (10^2 x^6 y^4)^{\frac{1}{2}} = 10x^3y^2$
 (d) $\sqrt[5]{64x^{12}y^{15}} = (2^6 x^{12} y^{15})^{\frac{1}{5}} = 2^{\frac{6}{5}} x^{\frac{12}{5}} y^3$

II. Evaluate each of the following without a calculator.

(a) $(27)^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$
 (b) $4^{2.5} = 4^{\frac{5}{2}} = \sqrt{4^5} = 2^5 = 32$
 (c) $(\frac{1}{16})^{\frac{3}{4}} = \frac{1}{\sqrt[4]{16^3}} = \frac{1}{2^3} = \frac{1}{8}$
 (d) $(-27)^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{(-27)^2}} = \frac{1}{(-3)^{-2}} = \frac{1}{9}$

LEVEL TWO: Rewrite the following as a single radical and simplify completely.

(a) $\sqrt{9x} \cdot \sqrt{4x^3} = \sqrt{36x^4} = 6x^2$
 (b) $\sqrt{7x^3} \cdot \sqrt{7x^3} = \sqrt{49x^6} = 7x^3$

(c) $\sqrt[3]{4x^2y} \cdot \sqrt[3]{2y^2} = \sqrt[3]{8x^2y^3} = 2y\sqrt[3]{x^2} = 2y x^{\frac{2}{3}}$
 (d) $2x\sqrt{3x^3} \cdot \sqrt{12x^5} = 2x\sqrt{36x^8} = 2x \cdot 6x^4 = 12x^5$

LEVEL THREE:

(a) Simplify the following expression:

$\sqrt{81x^2} \cdot (64x^3)^{-\frac{1}{3}}$
 $= (9x^2)^{\frac{1}{2}} \cdot (4^3 x^3)^{-\frac{1}{3}}$
 $= (9x)(4^{-1}x^{-1})$
 $= \frac{9x}{4x} = \frac{9}{4}$

(b) For $x > 0$, which expression is

equivalent to $\frac{\sqrt[3]{x^2} \cdot \sqrt{x^5}}{\sqrt[6]{x}}$? $(x^{\frac{2}{3}})(x^{\frac{5}{2}})$

(1) x

(2) $x^{\frac{3}{2}}$ (circled)

(3) x^3

(4) x^{10}

$x^{\frac{2}{3} + \frac{5}{2} - \frac{1}{6}} = x^{\frac{4}{6} + \frac{25}{6} - \frac{1}{6}} = x^{\frac{28}{6}} = x^{\frac{14}{3}}$

$\frac{x^{\frac{10}{6}}}{x^{\frac{1}{6}}} = x^{\frac{10}{6} - \frac{1}{6}} = x^{\frac{9}{6}} = x^{\frac{3}{2}}$
 subtract exponents

(c) The expression $\left(\frac{m^2}{m^3}\right)^{-\frac{1}{2}}$ is equivalent to

(1) $-\sqrt[6]{m^5}$

(2) $\frac{1}{\sqrt[6]{m^5}}$

(3) $-m\sqrt[5]{m}$

(4) $\frac{1}{m\sqrt[5]{m}}$

same base, subtract exponents

$$\left(m^{2-\frac{1}{3}}\right)^{-\frac{1}{2}} = \left(m^{\frac{5}{3}}\right)^{-\frac{1}{2}} = m^{-\frac{5}{6}} = \sqrt[6]{m^{-5}} = \frac{1}{\sqrt[6]{m^5}}$$

LEVEL FOUR:

(a) Explain why $81^{\frac{3}{4}}$ equals 27.

$81^{\frac{3}{4}}$ can be converted to $(3^4)^{\frac{3}{4}}$ by expressing 81 as 3^4 . We can use the power to power rule for exponents and multiply $(4)(\frac{3}{4})$ to result in 3. Now 3^3 can reduce to 27.
* you can also convert to radical form and reduce this way.

(b) Given the equal terms $\sqrt[3]{x^5}$ and $y^{\frac{5}{6}}$, determine and state y, in terms of x.

$$\sqrt[3]{x^5} = y^{\frac{5}{6}}$$

$\left(x^{\frac{5}{3}}\right)^{\frac{6}{5}} = \left(y^{\frac{5}{6}}\right)^{\frac{6}{5}}$ multiply by reciprocal to reduce to 1. solve for y

$$x^{\frac{30}{15}} = y^2 \Rightarrow \boxed{x^2 = y^2}$$

(c) Express the fraction $\frac{2x^{\frac{3}{2}}}{(16x^4)^{\frac{1}{4}}}$ in simplest radical form.

$$\frac{2x^{\frac{3}{2}}}{(2^4 x^4)^{\frac{1}{4}}} = \frac{2x^{\frac{3}{2}}}{2x^1} = x^{\frac{3}{2}-1} = x^{\frac{1}{2}} = \boxed{\sqrt{x}}$$

or $\frac{3}{2} - \frac{2}{2}$

Name _____

Rational Exponents & Radicals Exit Ticket