Topic/Aim: How do we evaluate limits approaching infinity? Friday, May 29th 2020

Objectives: By the end of the lesson, students will be able to:

- Evaluate limits approaching infinity.

Task (below): As you read and complete the task linked, please be sure to take notes. If you have any questions, please join Tomas at 1:00 PM or Katherine at 2:30 PM for office hours on google hangouts.

Video Reference:
https://www.youtube.com/watch?v=nViVR1rImUE

Rules for limits approaching infinity:

1) **Polynomial/Constant**

- The limit of a CONSTANT, as x approaches infinity or negative infinity, will be equal to that same constant number.

Example

1. Evaluate: $\lim_{x \to \infty} 2$ Solution: $\lim_{x \to \infty} 2 = 2$
2. Evaluate: $\lim_{x \to -\infty} 5$ Solution: $\lim_{x \to -\infty} 5 = 5$

- To find the limit of a POLYNOMIAL as x approaches infinity or negative infinity, identify the leading term (highest x power term) in the polynomial. Ignore all lower terms, because as x gets infinitely large (in either the positive or negative direction), the highest term is growing most quickly, and the lower terms will not affect the limit value. Then figure out whether this leading term will grow toward positive infinity or negative infinity, as x gets extremely large.

Example: Evaluate

1. $\lim_{x \to \infty} x^2 + 4x + 2$ Solution: $\lim_{x \to \infty} x^2 + \cancel{4}x + \cancel{2} = \infty$

2. $\lim_{x \to -\infty} x^2 + 4x + 2$ Solution: $\lim_{x \to -\infty} x^2 + \cancel{4}x + \cancel{2} = \infty$

3. $\lim_{x \to \infty} x^3 - 4$ Solution: $\lim_{x \to \infty} x^3 - \cancel{4} = \infty$

4. $\lim_{x \to -\infty} x^3 + 2x - 1$ Solution: $\lim_{x \to -\infty} x^3 + \cancel{2}x - \cancel{1} = -\infty$

2) Rational

- Steps:
  A. Identify the leading terms
  B. Ignore the lower terms
  C. Simplify the radical
  D. Apply the appropriate rule
     a. TOP HEAVY: If the degree of the numerator > the degree of the denominator, the limit Does Not Exist (ends approach $\mp\infty$)
     b. BOTTOM HEAVY: If the degree of the numerator < the degree of the denominator, the limit is equal to 0. (HA [horizontal asymptote] is y = 0)
     c. SAME DEGREE: If the degree of the numerator = the degree of the denominator, the limit is the ratio of the leading coefficients. (HA is y = ratio of the leading coefficients)

Example: Evaluate

1. $\lim_{x \to \infty} \frac{4x^3 + 2x - 7}{3x^2 + 2x - 5}$ Solution: Top Heavy 1. $\lim_{x \to \infty} \frac{x^3}{x^2} = DNE$

2. $\lim_{x \to \infty} \frac{2x^4 + 4x^3 - 3}{5x^5 - 4x - 1}$ Solution: Bottom Heavy 2. $\lim_{x \to \infty} \frac{x^4}{x^5} = 0$

3. $\lim_{x \to \infty} \frac{8x^3 - 2x^2 + 1}{2x^3 - 4x - 7}$ Solution: Same Degree 3. $\lim_{x \to \infty} \frac{8x^3}{2x^3} = \frac{8}{2} = 4$

3) Exponential

Example: Evaluate $\lim_{x \to \infty} e^{-2x}$

First rewrite the expression e^{-2x} using the reciprocal function, so $\frac{1}{e^{2x}}$. It is easier to see what happens as x gets extremely large and goes toward infinity. The e^{2x} gets extremely large, so 1 over a very large number will head toward zero, and the limit will be equal to 0.

$$\lim_{x \to \infty} e^{-2x} = 0$$

PRACTICE: Evaluate each limit.

1. $\lim_{x \to -\infty} \left(x^3 - 4x^2 + 5 \right)$

2. $\lim_{x \to -\infty} -\left(2x^5 + 3x^3 - x - 4 \right)$

3. $\lim_{x \to -\infty} \frac{3x^2 + 4x + 3}{x^3 + x + 14}$

4. $\lim_{x \to \infty} \frac{2x^2 + 6}{3x^2 - 4x + 1}$

5. $\lim_{x \to \infty} \frac{2x^2 - 3x + 5}{x - 2}$

6. $\lim_{x \to -\infty} \frac{\sqrt{2x^2 + 3}}{2x + 3}$

7. $\lim_{x \to -\infty} \frac{\sqrt{2x^2 + 1}}{4x + 2}$

8. $\lim_{x \to \infty} \left(e^x - 3 \right)$

9. $\lim_{x \to -\infty} -e^{-4x}$

10. $\lim_{x \to \infty} \left(-e^{-3x} - 1 \right)$