Mathematician ______________________ STATION 1: Writing Exponential Equations

Exponential Formula Type	Exponential Equation	Compounding	Continuously Compounding
Formulas	$A = a_0 (1 \pm r)^t$	$A = P(1 + \frac{r}{n})^{nt}$	$A = Pe^{rt}$
When to use it/helpful info.	Growth → + 1 Decay → - 1	When we see the word compounding, this is n. Monthly → n = 12 Quarterly → n = 4	Whenever you see the word *continuous*

PRACTICE:

1. For each example below, identify the initial amount and rate of decay (as a percent):

(a) $g(t) = 240\,(0.75)^t$

25% decay

initial : 240

(b) $h(t) = 175(1.028)^t$

initial 2.8%

(c) $k(t) = 6.72(2)^t$

initial 100%

$\boxed{\text{(d)}}$ $y = (\frac{7}{8})^t$

decay: 1/8 12.5%

initial : 1

2. The growth rate of a bacterial culture is 150% each hour. Initially, there are 10 bacteria. Describe and correct the error in finding the number of bacteria in the culture after 8 hours.

1 + 1.5 = 2.5

$b(t) = 10(1.5)^t$

$b(8) = 10(1.5)^8 \approx 256.3$

After 8 hours, there are about 256 bacteria in the culture.

t = 5 r = .055 5.5 n = 4

4. After 5 years of interested payments of $5\frac{1}{2}$% compounded quarterly, an account has \$5046.02. What is the principal?

P? A

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$5046.02 = P\left(1 + \frac{.055}{4}\right)^{(4)(5)}$$

$$\frac{5046.02}{(1.01375^{20})} = \frac{P(1.01375)^{20}}{1.01375^{20}}$$

$$\$3840.00 = P$$

5. The inaugural attendance of an annual music festival is 150,000. The attendance y increases by 8% each year.

(a) Write an exponential growth function that represents the attendance after t years.

$f(t) = 150{,}000(1+.08)^t$

(b) How many people will attend the festival in the fifth year? Round your answer to the nearest thousand.

$f(5) = 150{,}000(1.08)^5 = 220{,}399.2115$

220,399 people

6. The value of a car is $21,500. It loses 12% of its value every year.

(a) Write a function that represents the value y (in dollars) of the car after t years.

$y = 21{,}500(1-.12)^t$

(b) What will be the value of the car after 6 years.

$y = 21{,}500(.88)^6$ $9,984.69

7. A website has 500,000 members in 2010. The number y of members increases by 15% each year.

(a) Write an exponential growth function that represents the website membership *t* years after 2010.

$y = 500{,}000(1+.15)^t$

(b) How many members will there be in 2016? Round your answer to the nearest ten thousand.

$y = 500{,}000(1.15)^6 = 1{,}156{,}530.383$

1,156,530 members

8. The yearly growth of a new population of gnats can be modeled by the equation $G(t) = 500(1.182)^t$, where G(t) is the number of gnats, *t* is the time in years, and 500 is the starting population. What is the *monthly growth rate* of these gnats?

$1.182 = \cancel{b} + r$

-1

$.182 = r$

$\frac{.182}{12}$

$.015167 =$ monthly rate

or

1.5%

Mathematician ______________________ STATION 2: Solving Expntl + Log Equations

What does the problem look like?	Logs are on both sides	Log on one side	Exponents	Exponent
What do I do?	Condense with rules (if needed) and drop the logs when equation is log x = log y.	Isolate log term and convert to exponential.	Make same base on each side then solve equation from exponents.	Add logs to both sides, condense, and convert if needed.

Some other things not to forget:

- Ln = $\log_e$
- e has it's own button in the calculator
- $x^{\frac{p}{r}} = \sqrt[r]{x}^p$

Solve for x:

(a) $2^x = 30$

$\log 2^x = \log 30$

$\frac{x \log 2}{\log 2} = \frac{\log 30}{\log 2}$

$x = 4.91$

(b) $\log_4 x + \log_4 (x - 12) = 3$

$\log_4(x^2 - 12x) = 3$

$4^3 = x^2 - 12x$

$0 = x^2 - 12x - 64$

$0 = (x-16)(x+4)$

$\boxed{x = 16}$

$x = -4$ reject

(c) $\log_4 (2x + 1) = \log_4 (x + 2) - \log_4 3$

$\cancel{\log_4}(2x+1) = \cancel{\log_4}\left(\frac{x+2}{3}\right)$

$\frac{2x+1}{1} = \frac{x+2}{3}$

$x + 2 = 6x + 3$

$-x \; -3 \quad -x \; -3$

$\frac{-1}{5} = \frac{5x}{5}$

$x = \frac{-1}{5}$

(d) $3^{2x-1} = 27$

$3^{2x-1} = 3^3$

$2x - 1 = 3$

$+1 \quad +1$

$\frac{2x}{2} = \frac{4}{2}$

$\boxed{x = 2}$

* (e) $3.4e^{2-2n} - 9 = -4$

$+9 \quad +9$

$\frac{3.4e^{2-2n}}{3.4} = \frac{5}{3.4}$

$e^{2-2n} = \frac{5}{3.4}$

Ln

$\log_e\left(\frac{5}{3.4}\right) = 2 - 2n$

$0.3857 = 2 - 2n$

$-2 \quad -2$

$\frac{-1.6143}{-2} = \frac{-2n}{-2}$

$.807 = n$

* (f) $8^{x-2} = \sqrt{8}$

$8^{x-2} = 8^{\frac{1}{2}}$

$x - 2 = \frac{1}{2}$

$+2 \quad +2$

$\boxed{x = 2\frac{1}{2}}$

(g) $\log(x - 1) + \log 3 = \log x$

$\log(3x-3) = \log x$

$3x - 3 = x$

$-3x \quad -3x$

$\frac{-3}{-2} = \frac{-2x}{-2}$

$\boxed{x = \frac{3}{2}}$

(h) $2\log 5 + \frac{1}{2}\log 9 - \log 3 = \log x$

$\log 25 + \cancel{\log 3} - \cancel{\log 3} = \log x$

$\log 25 = \log x$

$\boxed{25 = x}$

(i) $\frac{250(1.04)^x}{250} = \frac{1000}{250}$

$\log 1.04^x \ \log 4$

$\frac{x \log 1.04}{\log 1.04} = \frac{\log 4}{\log 1.04}$

$x = 35.35$

(j) $\ln x + \ln(x + 1) = 5$

$\ln(x^2+x) = 5$

$\log_e(x^2+x) = 5$

$e^5 = x^2+x$

not fun to solve ӧ

(k) $4^{x+1} = \frac{1}{64}$

$4^{x+1} = 4^{-3}$

$x+1 = -3$

$-1 \quad -1$

$\boxed{x = -4}$

(l) $\log 2\ \frac{x}{3} = 4$

mistake
base was
supposed to be 2

Solving Equations from Word Problems (review writing exponential equations before doing this):

1. The population of gnats increases yearly at a constant rate. In one year, the population *increases* from 600 to 680. Find the yearly growth rate(to nearest tenth) and express the yearly growth factor.

2. A college fund is started for Ashton on his fifth birthday. The initial investment of $2500 is compounded bimonthly at a rate of 9%. How old will Ashton be when the account balance has quadrupled?

3. A colony of bacteria grows according to the equation $N(t) = 100e^{0.1t}$, where N is measured in grams and t is measured in days.

(a) Determine the initial amount of bacteria and the growth rate of the bacteria?

(c) How long does it take for the population to triple?

Mathematician ______________________ STATION 3: Sequences + Series

	Arithmetic	Geometric
Recursive	$a_n = a_{n-1}\ d$	$a_n = a_{n-1}\ r$
Explicit	$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}$
Summation	$S_n = \frac{n}{2}(a_1 + a_n)$	$S_n = \frac{a_1 - a_1 r^n}{1-r}$

1. Write the sum without using summation notation, and find the sum.

(a) $\sum_{m=0}^{6} 2m$

$2(0)+2(1)+2(2)+2(3)+2(4)+2(5)+2(6) = 42$

$0+2+4+6+8+10+12$

* $\boxed{(b)}$ $\sum_{n=1}^{5} (-1)^{n+1}(2n)$

$(-1)^2(2)+(-1)^3(4)+(-1)^4(6)+(-1)^5(8)+(-1)^6(10)$

$+2-4+6-8+10 = \boxed{6}$

2. Express the following sums using sigma notation and find the sum:

(a) 7 + 14 + 21 + 28 + … + 105

$7(1)+7(2)+7(3)+7(4)+\dots+7(15)$

$\sum_{n=1}^{15} 7n$

$\boxed{(b)}$ $\frac{5}{6+3} + \frac{5}{7+3} + \frac{5}{8+3} + \dots + \frac{5}{31+3}$

$\frac{5}{\ \ +3}\quad \frac{5}{\ \ +3}\quad \frac{5}{\ \ +3}\ \dots\ \frac{5}{\ \ +3}$

adding by 1

$\boxed{\sum_{n=6}^{31} \frac{5}{n+3}}$

3. The first term of an arithmetic sequence is equal to 6 and the common difference is equal to 3. Find value of the 50th term.

$a_1 = 6$ $d = 3$

$a_{50} = ?$ $a_n = (n-1)(d) + a_1$

$a_{50} = (50-1)(3) + 6 = \boxed{153}$

4. An arithmetic sequence has a its 5th term equal to 22 and its 15th term equal to 62. Find its 100th term.

Need to find d

$a_5 = 22$ $n = 5$

$a_{15} = 62$ $a_{100} = ?$

$22 = (5-1)d + a_1$

$62 = (15-1)d + a_1$

$22 = 4d + a_1$

$-\ 62 = 14d + a_1$

$-40 = -10d$

$d = 4$

$22 = (5-1)(4) + a_1$

$22 = 16 + a_1$

$6 = a_1$

$a_{100} = (100-1)(4) + a_1$

$(99)(4) + 6$

$\boxed{a_{100} = 402}$

$f(2)$ $f(3)$

5. Find the number of terms in the geometric progression 6, 12, 24, ..., 1536

Solving for n when $f(n) = 1536$ $r = 2$

$f(n) = (n-1) \cdot r$

$1536 = (n-1)2$

$1536 = 2n - 2$

$+2 \quad +2$

$\frac{1538}{2} = \frac{2n}{2}$

$\boxed{769 = n}$

6. A construction company will be penalized each day of delay in construction for bridge. The penalty will be \$4000 for the first day and will increase by \$10,000 for each following day. Based on its budget, the company can afford to pay a maximum of \$ 165,000 toward penalty. Find the maximum number of days by which the completion of work can be delayed.

arithmetic

$d = 10{,}000$

$a_1 = 4{,}000$

$165{,}000 = (n-1)(10{,}000) + 4{,}000$

$165{,}000 = 10{,}000n - 10{,}000 + 4{,}000$

$165{,}000 = 10{,}000n - 6{,}000$

$+6{,}000 \quad +6{,}000$

$\frac{171{,}000}{10{,}000} = \frac{10{,}000n}{10{,}000}$

$17.1 = n$

the company can work for 17 days

7. A video of a grumpy cat has gone viral. On the first day, the video was viewed 6,000 times. Each day the number of daily visits is increasing by 40%. Find the cumulative number of views over the first two weeks.

a_1

14 days

rate 1.4

Geometric

adding/sum

$$S_{14} = \frac{6{,}000 - 6{,}000(1.4)^{14}}{1 - 1.4}$$

$S_{14} = 1651801.024$

1,651,801 views