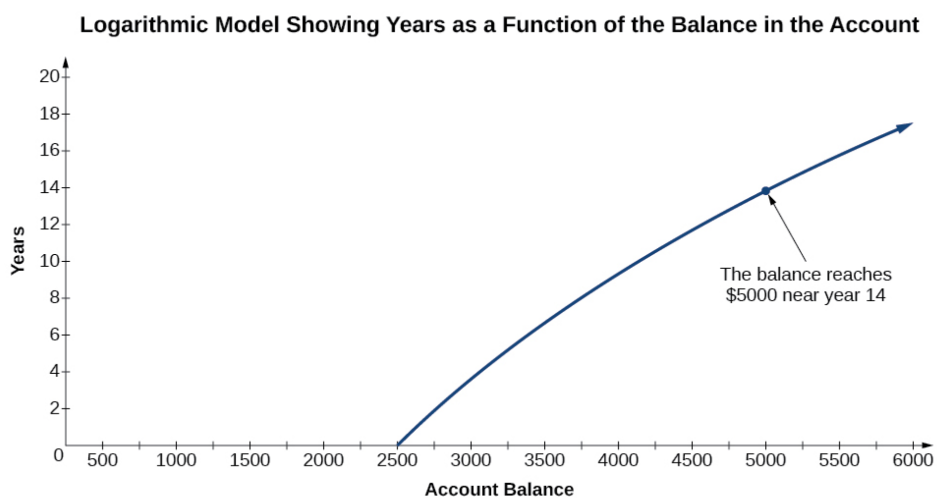
TASK: Please read and complete the following task. As you read through, be sure to take down any important facts in your notes.

In Graphs of Exponential Functions, we saw how creating a graphical representation of an exponential model gives us another layer of insight for predicting future events. How do logarithmic graphs give us insight into situations? Because every logarithmic function is the inverse function of an exponential function, we can think of every output on a logarithmic graph as the input for the corresponding inverse exponential equation. In other words, logarithms give the ***cause*** for an ***effect***.

To illustrate, suppose we invest $2500 in an account that offers an annual interest rate of 5%, compounded continuously. We already know that the balance in our account for any year ***t*** can be found with the equation $A = 2500e^{0.05t}$.

But what if we wanted to know the year for any balance? We would need to create a corresponding new function by interchanging the input and the output; thus we would need to create a logarithmic model for this situation. By graphing the model, we can see the output (year) for any input (account balance). For instance, what if we wanted to know how many years it would take for our initial investment to double? The logarithmic graph below shows this point.



Finding the Domain of a Logarithmic Function: Before working with graphs, we will take a look at the domain (the set of input values) for which the logarithmic function is defined. Recall that the exponential function is defined as $y = b^x$ for any real number x and constant $b > 0$ $b \neq 1$, where

- The domain of y is $(-\infty,\infty)$
- The range of y is $(0,\infty)$.

We also know that the logarithmic function $y = \log_b(x)$ is the inverse of the exponential function $y = b^x$. So, as inverse functions:

- The domain of $y = \log_b(x)$ is the range of $y = b^x$: $(0,\infty)$
- The range of $y = \log_b(x)$ is the range of $y = b^x$: $(-\infty,\infty)$

Transformations of the parent function $y = \log_b(x)$ behave similarly to those of other functions. Just as with other parent functions, we can apply the four types of transformations—shifts, stretches, compressions, and reflections—to the parent function without loss of shape.

For example, consider f(***x***) = $\log_4(2x-3)$. This function is defined for any values of *x* such that the argument, in this case $2x-3$,is greater than zero. To find the domain, we set up an inequality and solve for *x*:

$2x - 3 > 0$ Greater than zero because we can't take the log of negatives.

$2x > 3$ Add 3

$x > 1.5$ Divide by 2

In interval notation, the domain of $f(x) = \log_4(2x - 3)$ is $(1.5, \infty)$.

To summarize the work above, we can create a set of steps to identify the domain of a logarithmic function:

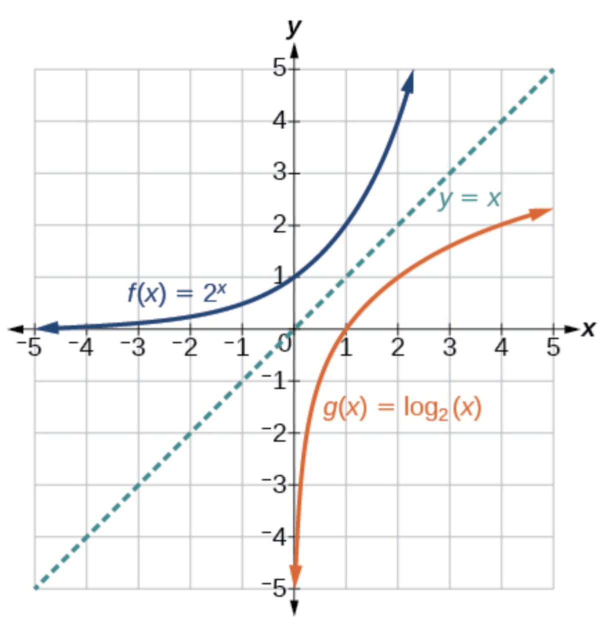
1. Set up an inequality showing the argument greater than zero.
2. Solve for x.
3. Write the domain in interval notation.

Now you try: Identify the domain for the following logarithmic functions.

(a) $f(x) = log_2(x + 3)$

(b) $g(x) = log_5(x - 2) + 1$

Graphing a Logarithmic Function: Before graphing logarithmic functions, recall we can easily graph exponential functions and use facts about inverse functions as well as key function features. The functions ***f***(***x***) = 2^x and ***g***(***x***) = $\log_2 x$ are graphed below. What special coordinates do you notice on the graph? How do they relate to the functions?



To graph logarithmic functions with the form $f(x) = \log_b(x)$, we can follow the steps:

1. Draw and label the vertical asymptote, x = 0.
2. Plot the x-intercept (1, 0).
3. Plot the key point (***b***, 1).
4. Draw a smooth curve through the points.
5. State the domain, $(0, \infty)$, the range $(-\infty, \infty)$, and the vertical asymptote, x = 0.

Now you try:

(a) Graph $f(x) = \log_5(x)$. State the domain, range, and asymptote.

(b) Sketch the horizontal shift $f(x) = \log_3(x - 2)$ alongside its parent function. State the domain, range, and asymptote.