The female honeybee is produced after the queen mates with a male so the female has two parents, a male and a female. The male honeybee, however, is produced by the queen's unfertilized eggs and thus has only one parent, a female. The family tree for the honeybee follows a special sequence.

Generation	1	2	3	4	5	6
Ancestors	1	1	2	3	5	8

Notice that every term in the list of ancestors is the sum of the previous two terms. This special sequence is called the ***Fibonacci sequence***, and it is found in many places in nature (have a free minute? Google it!). The Fibonacci sequence is an example of a ***recursive sequence***. In a recursive sequence, each term is determined by one or more of the previous terms.

Sorry, we have more formulas! The formulas you used for sequences thus far have been explicit formulas. An ***explicit formula*** gives a_n as a function of *n*, the formula that describes the Fibonacci sequence, uses a_{n-1}. This is an example of a ***recursive formula***, which means that every term will be determined by one or more of the previous terms. An initial term must be given in a recursive formula.

Recursive Arithmetic Sequence	Recursive Geometric Sequence
$a_n = a_{n-1} + d$	$a_n = a_{n-1} \cdot r$

<u>Examples:</u> Write a recursive formula/rule for each sequence below.

(1) 2, 10, 18, 26, 34, …

Step 1 Determine whether the sequence is arithmetic or geometric.
The sequence is arithmetic because each term after the first can be found by adding a common difference.

Step 2 Find the common difference.

$d = 10 - 2$ or 8

Step 3 Write the recursive formula.

$a_n = a_{n-1} + d$ Recursive formula for arithmetic sequence

$a_n = a_{n-1} + 8$ $d = 8$

A recursive formula for the sequence is $a_n = a_{n-1} + 8, a_1 = 2$.

(2) 16, 56, 196, 686, 2401, …

Step 1 Determine whether the sequence is arithmetic or geometric.
The sequence is geometric because each term after the first can be found after multiplying by a common ratio.

Step 2 Find the common ratio.

$r = \frac{56}{16}$ or 3.5

Step 3 Write the recursive formula.

$a_n = r \cdot a_{n-1}$ Recursive formula for geometric sequence

$a_n = 3.5a_{n-1}$ $r = 3.5$

A recursive formula for the sequence is $a_n = 3.5a_{n-1}, a_1 = 16$.

1. Write a recursive formula for each sequence below:

(a) 3, 8, 18, 38, 78, …

(b) 8, 17, 26, 35, 44, …

(c) $a_3 = 16$ and $r = 4$

2. Mr. Edwards and his company deposit $20,000 into his retirement account at the end of each year. The account earns 8% interest before each deposit.

(a) Write a recursive formula for the balance in the account at the end of each year.

(b) Determine how much is in the account at the end of each of the first 8 years.

3. Find the first five terms of the sequence in which $a_1 = -3$ and $a_{n+1} = 4a_n - 2$, if $n \geq 1$.

4. Solve the following two systems of equations.

(a)
$$x + 2y - z = -4$$
$$-2x + 4y - z = 6$$
$$2x + 2y + 3z = 1$$

(b)
$$2y + 6 = x$$
$$y^2 - 9 = x$$