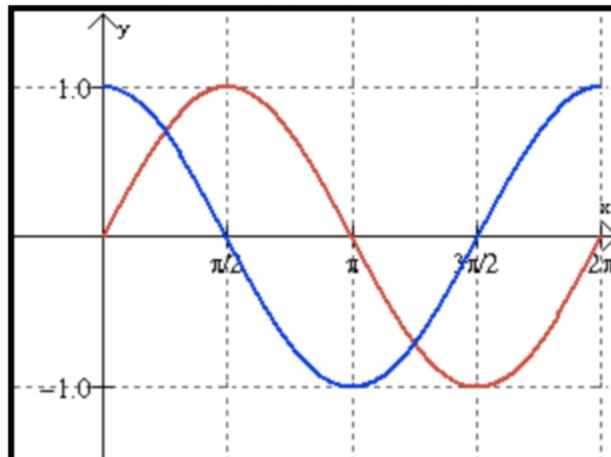


EXPLORE: We have noticed throughout this unit that the Cosine and Sine curve look almost identical, if we performed a *horizontal* shift.

The graphs to the right can be represented with the equations $y = \cos(x)$ and $y = \sin(x)$.

We actually can use cosine to represent both equations or sine to represent both equations if we take the horizontal shifts into account. Each curve actually has an infinite of equations to be used since the periodic function will continue to intersect the x axis over and over and the function continues to infinity.

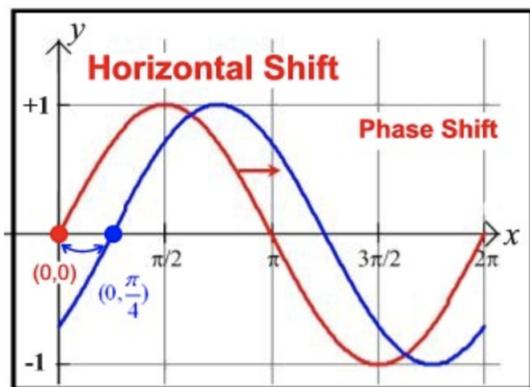


Horizontal Shift 1: We can visualize shifting the $y = \cos(x)$ curve to the right $\frac{\pi}{2}$ so that the two graphs appear to overlap. This would allow the red graph to now be expressed as $y = \cos(x - \frac{\pi}{2})$. Since we moved to the right, the transformation is subtracted.

Horizontal Shift 2: We can visualize shifting the $y = \sin(x)$ curve to the left $\frac{\pi}{2}$ so that the two graphs appear to overlap. This would allow the blue graph to now be expressed as $y = \sin(x + \frac{\pi}{2})$. Since we moved to the left, the transformation has been added.

What is the big takeaway from today's lesson?

We can create an infinite number of equations for a trig graphs by analyzing the horizontal shifts (h), by adding to the left and subtracting to the right. The shift (h) is the horizontal distance from the x-intercept to the y-axis (where we normally “start” our trig functions).

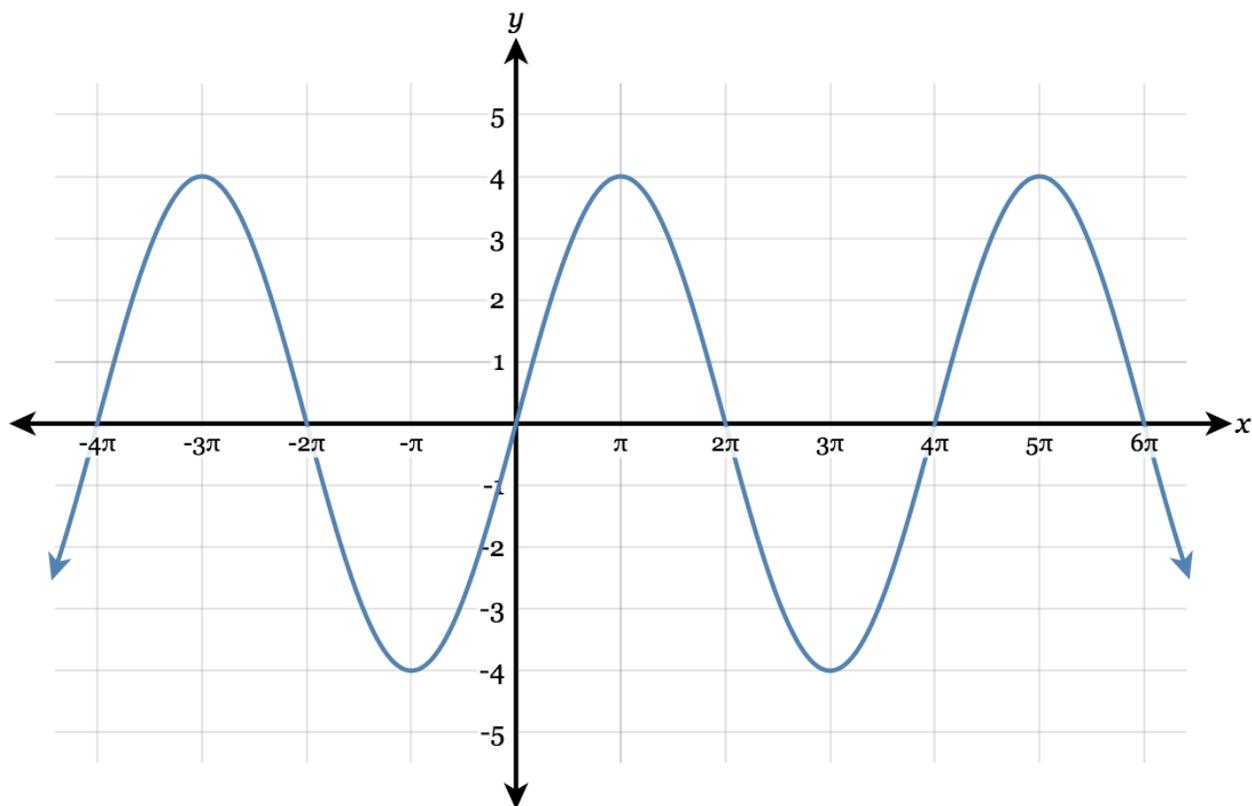


$$y = \sin(x) \quad y = \sin\left(x - \frac{\pi}{4}\right)$$

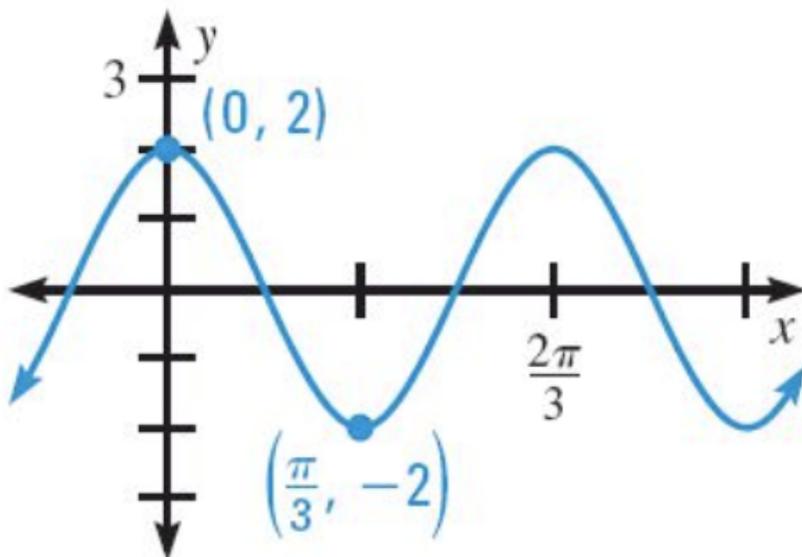
*Sometimes we cannot see how much the shift is on the x-axis. It is safe to assume we are splitting the value in half. For example, the red graph looks like it shifts to the right $\frac{\pi}{4}$, we found this value by dividing $\frac{\pi}{2}$ by 2 or the same as multiplying by the reciprocal $\frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$.

PRACTICE: Determine a Sine and Cosine equation for the functions graphed below.

1. **Only the Cosine curve will have a horizontal shift here.*

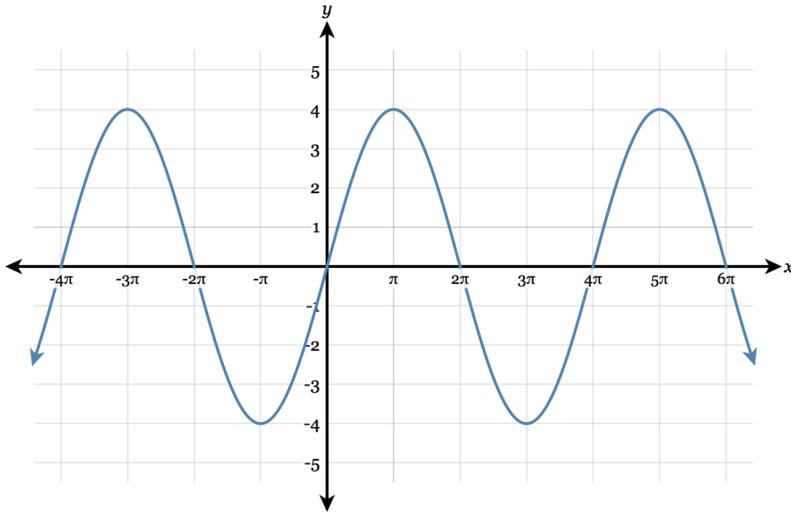


2. **Only the Sine curve will have a horizontal shift here.*



ANSWER KEY: Determine a Sine and Cosine equation for the functions graphed below.

1.



*This originally looks like a Positive Sine graph. So the Sine curve will have a horizontal shift of zero because it intersects the y-axis at the midline.

a: 4

m: $y = 0$

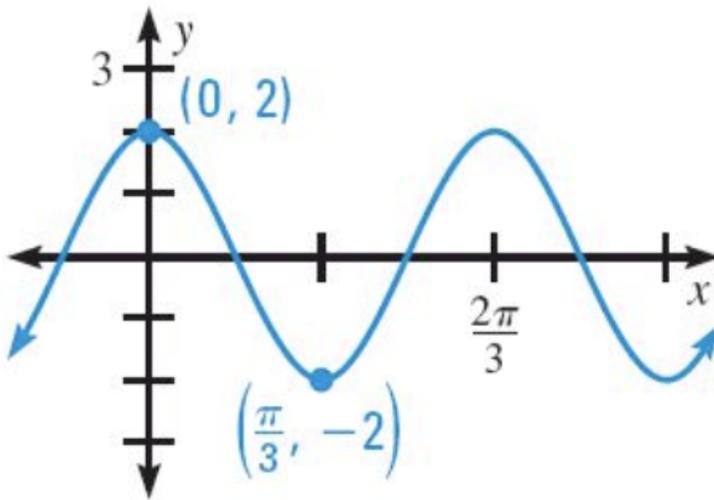
f: $\frac{1}{2}$

$$y = 4 \sin\left(\frac{1}{2}x\right) + 0$$

Cosine curves intersect the y-axis at the maximum or minimum. I see the graph hit the maximum at 4 a distance of π to the right. So the Cosine equation is:

$$y = 4 \cos\left(\frac{1}{2}x - \pi\right) + 0$$

2.



*This originally looks like a Positive Cosine graph. So the Cosine curve will have a horizontal shift of zero because it intersects the y-axis at the maximum.

a: 2

m: $y = 0$

P: $\frac{2\pi}{3} \rightarrow f\left(\frac{2\pi}{3}\right) = 2\pi$

f: 3

$$y = 2 \cos(3x) + 0$$

Sine curves intersect the y-axis at the midline. The tick marks are progression by $\frac{\pi}{3}$ and it looks like the graph intersects the midline halfway through at $\frac{\pi}{6}$. So the sine equation is:

$$y = 2 \sin\left(3x - \frac{\pi}{6}\right) + 0$$