Topic/Aim: An Introduction to Limits Tuesday, May 26th 2020

Wrapping up our year in Precalculus, we will be spending the last two weeks of classes preparing for Calculus. One of the biggest concepts to help you prepare for Calculus, is to learn about **LIMITS**.

EXPLORE: Jacinda has a 1988 Rustang that she wants to sell. Travis is interested in buying her car, but they have not decided on a price. Travis offers $1000 for the car stating that this is what the car is worth according to its Blue Book value. Jacinda states, "*My car is worth more than $1000! If I wanted the Blue Book value, I would have traded it in when I bought my new car. If you look at the used cars advertised in the classified ads, you will see it is worth a lot more than $1000*."

Taking on the challenge, Travis agrees to look at similar Rustangs in the classified section of the newspaper. Below are all the Rustangs that Travis finds advertised.

Year	1978	1980	1981	1983	1984	1986
Asking Price	$900	$1220	$1380	$1700	$1860	$2180

A. From the data, can you make a prediction about the asking price for a 1988 Rustang? How reliable is this prediction?

B. Jacinda decides to do her own investigation using a local paper. Below is her data. According to her research, what price do you predict for the 1988 Rustang?

Year	1990	1991	1993	1994	1996
Asking Price	$4450	$5125	$6475	$7150	$8500

C. Based on this information, will Travis and Jacinda agree on the price?

D. Jacinda and Travis decide that additional research is necessary. They grab another paper and find a 1987 Rustang for sale for $2340 and a 1989 Rustang for sale for $3775. Will this new information help them to make a decision about the fair price of the car?

By trying to predict the price for the 1988 Rustang, we are seeking a "**limit**," or a final prediction of the price as the year approaches 1988. This can be written:

$$\lim_{t \to 1988-} (asking\ price) = \$2500 \quad \text{and} \quad \lim_{t \to 1988+} (asking\ price) = \$3100$$

E. The left-hand limit is read, "*As the year approaches 1988 from the left, the asking price approaches $2500.*" Translate the right-hand limit into a sentence:

F. When we write the limit without a plus or minus sign, we are using both sides of 1988 to estimate a value: $\lim_{t \to 1988} (asking\ price)$. Since $\lim_{t \to 1988-} (asking\ price) \neq \lim_{t \to 1988+} (asking\ price)$, we state that $\lim_{t \to 1988} (asking\ price)$ ***does not exist*** because the two sides do not agree.

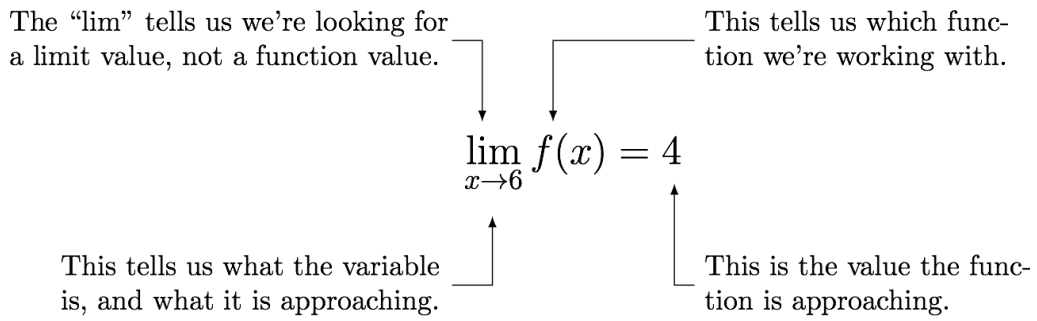
- - - - - - - - - - - - - - Copy the following down in your notes - - - - - - - - - - - - - -

$$\lim_{x \to a} f(x) = L$$

The limit of the function ***f(x)*** as x ***approaches*** a, equals the constant L.

Note: The value of a limit depends only on the functions behaviour near the value of "a", NOT AT "a".

NOTATION:



ONE SIDED LIMITS

- One-sided limits can be used to show that a function has a limit as x approaches a.

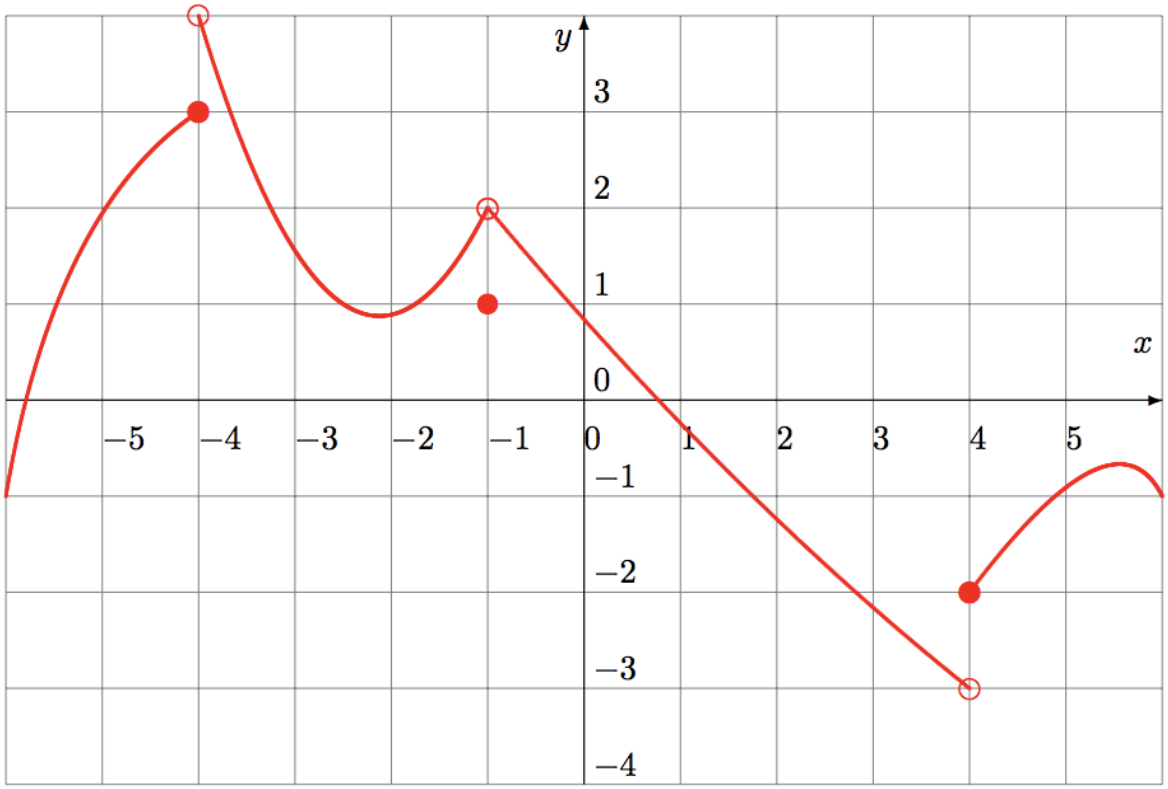
$$\lim_{x \to a} f(x) = L \text{ if and only if both}$$
$$\lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L.$$

- One-sided limits can be used to show that a function has no limit as x approaches a.

$$\text{If } \lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = M, \text{ where } L \neq M,$$
$$\lim_{x \to a} f(x) \text{ does not exist.}$$

MODEL EXAMPLE:

1. Consider the following function defined by its graph:



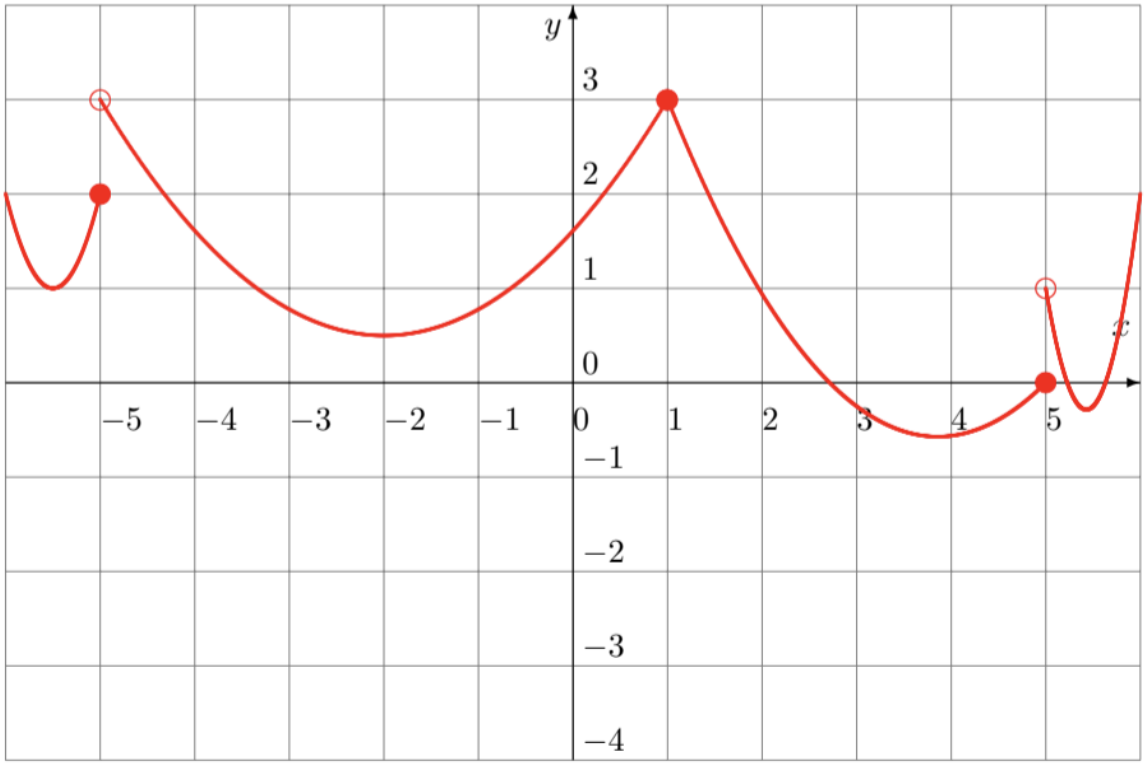
Find the following limits:

a) $\lim_{x \to -1^-} f(x)$ b) $\lim_{x \to -1^+} f(x)$ c) $\lim_{x \to -1} f(x)$ d) $\lim_{x \to -4} f(x)$ e) $\lim_{x \to 4} f(x)$

[WATCH/LISTEN TO KATHERINE EXPLAIN THE ANSWERS TO THESE QUESTIONS]

NOW YOU TRY:

2. Consider the following function defined by its graph:



Find the following limits:

a) $\lim_{x \to 1^-} f(x)$ b) $\lim_{x \to 1^+} f(x)$ c) $\lim_{x \to 1} f(x)$ d) $\lim_{x \to -5} f(x)$ e) $\lim_{x \to 5} f(x)$