Limits Approaching Infinity Special Cases: Rational & Radical Functions

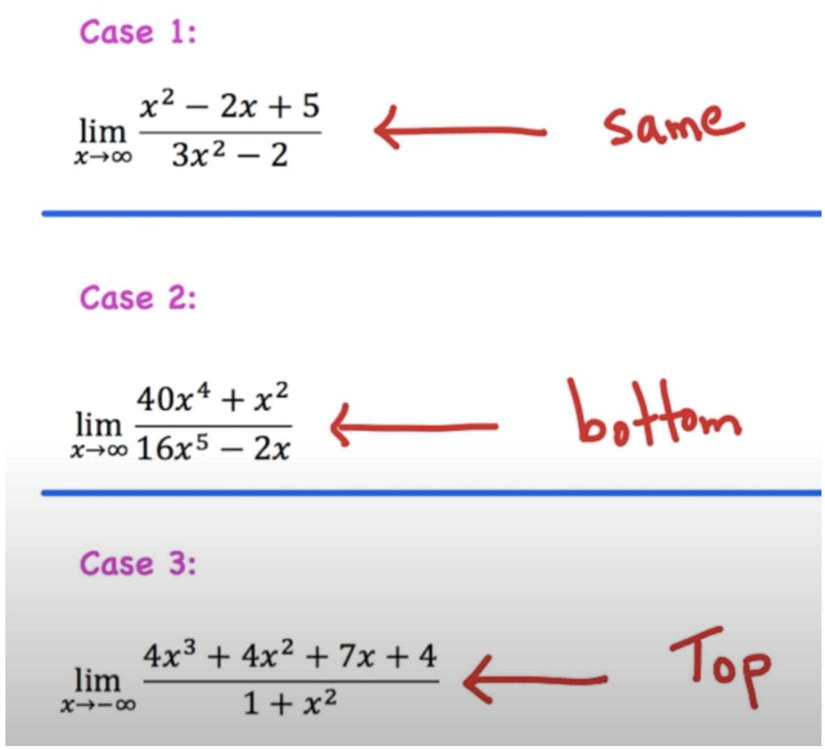
Steps:

(a) Identify the leading terms and ignore the lower terms.

(b) Simplify the radical (if there is one)

(c) Apply the appropriate rule: If Degree of …

1. Numerator > Denominator, then lim approaches $\pm\infty$
2. Numerator < Denominator, then lim = 0
3. Numerator = Denominator, then lim = ratio of the Leading Coefficient



[Want to watch a video of Katherine explaining an example before moving on? CLICK HERE!]

Model Example: $\lim_{x \to \infty} \frac{\sqrt{x^2 - 3x}}{2x + 1} \to \frac{\sqrt{x^2}}{2x} \to \frac{x}{2x} \to \frac{1}{2}$

[since the degree was the same, I found the ratio of the leading terms.]

Practice: Evaluate the following limits.

$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+4}}$	$\lim_{x\to\infty}\left(\frac{3x-7}{5x^4-8x+12}\right)$
$\lim_{x\to\infty}\left(\frac{x^6-2}{10x^4-9x+8}\right)$	$\lim_{x\to\infty} \frac{\sqrt[3]{-39+30x^6-27x^8}}{2x^2+1}$
$\lim_{x\to\infty}\left(-1+x+x^4\right)$	$\lim_{x\to-\infty} \frac{\sqrt{16x^4+16x^2}}{6x^2-1}$

Limits Approaching Infinity Special Cases: Working with *e*

We can simplify or factor expressions and use the following information about limits with *e*.

$$\lim_{x\to+\infty} e^x = +\infty \qquad \lim_{x\to-\infty} e^x = 0$$

Model Example: $\lim_{x\to\infty} \frac{6\mathbf{e}^{4x}-\mathbf{e}^{-2x}}{8\mathbf{e}^{4x}-\mathbf{e}^{2x}+3\mathbf{e}^{-x}}$

$$\begin{aligned}\lim_{x\to\infty} \frac{6\mathbf{e}^{4x}-\mathbf{e}^{-2x}}{8\mathbf{e}^{4x}-\mathbf{e}^{2x}+3\mathbf{e}^{-x}} &= \lim_{x\to\infty} \frac{\mathbf{e}^{4x}\left(6-\mathbf{e}^{-6x}\right)}{\mathbf{e}^{4x}\left(8-\mathbf{e}^{-2x}+3\mathbf{e}^{-5x}\right)} \\ &= \lim_{x\to\infty} \frac{6-\mathbf{e}^{-6x}}{8-\mathbf{e}^{-2x}+3\mathbf{e}^{-5x}} \\ &= \frac{6-0}{8-0+0} \\ &= \frac{3}{4}\end{aligned}$$

Practice: Evaluate the following limits.

$\lim_{x \to \infty} \left(x^3 + 6x^2\right) e^{-x}$	$\lim_{x \to -\infty} \dfrac{3x^2 + 6x^3}{3x^3 - e^{-x}}$