3 Types of Systems of Equations:

Type of System/What do they look like?	How can I answer it? What steps can I use?	Sample Problems
Three equations with three variables	1. Make sure all equations are in standard form $Ax + By + Cz = D$. (all variables on left in order and constant on right) 2. Decide which variable to eliminate, coefficients need to be the same for this variable. If coefficients are not the same, multiply equations to make coefficients the same. It is helpful when one is negative. 3. Combine two equations to eliminate variable. Result will be a second equation with two variables. 4. Pick a different combination of two equations and eliminate the same variable again. 5. Solve new system of two equations you just made. 6. Using the solution for the two variables, substitute the values into one of the original equations and solve for the third variable.	$4x + 2y - 2z = 10$ $2x + 8y + 4z = 32$ $30x + 12y - 4z = 24$ (answer on left of page 2) $x - 3y + 3z = -4$ $2x + 3y - z = 15$ $4x - 3y - z = 19$ (answer on right of page 2)
Two nonlinear equations to solve algebraically • We cannot easily get both into "y = " or "x = " form. • There is frequently a "y^2".	1. Solve one equation for one variable in terms of the other. 2. Substitute the expression from Step 1 into the other equation. 3. Solve the resulting equation, frequently this is a Quadratic with two solutions. 4. Substitute solution(s) back into the other original equation. You may have to do this for both solutions.	$y = x^2 - x - 6$ $y = 2x - 2$ (answer on left of page 3) $x^2 + y^2 = 10$ $y = x + 2$ (answer on right of page 2)
Two nonlinear equations to solve with a graphing calculator • Can easily be in "y=" form	1. Are both equations in "y =" form? If not, manipulate so that it is. 2. Enter each function in the "y =" in your graphing calculator. 3. Use the TRACE function to determine the INTERSECTION of the functions. *If you don't have your graphing calculator - use www.Desmos.com	$f(x) = 3\|x\| - 1$ $g(x) = 0.03x^3 - x + 1$

$4x + 2y - 2z = 10$

$2x + 8y + 4z = 32$

$30x + 12y - 4z = 24$

Multiply equation by 2 so we can eliminate Z

$2 \cdot (4x + 2y - 2z = 10)$

$8x + 4y - 4z = 20$

1) Pair equations to eliminate Z

$$\begin{array}{r} 8x + 4y - 4z = 20 \\ 2x + 8y + 4z = 32 \\ \hline 10x + 12y = 52 \end{array} \qquad \begin{array}{r} 2x + 8y + 4z = 32 \\ 30x + 12y - 4z = 24 \\ \hline 32x + 20y = 56 \end{array}$$

2) Solve new system

$32 \cdot 10x + 12y = 52$

$10 \cdot 32x + 20y = 56$

$$\begin{array}{r} 320x + 384y = 1664 \\ -\ (320x + 200y = 560) \\ \hline 184y = 1104 \\ y = 6 \end{array}$$

Substitute Y

$10x + 12y = 52$

$10x + 12(6) = 52$

$x = -2$

Substitute x and y to find Z

$8x + 4y \quad 4z = 20$

$8(-2) + 4(6) - 4z = 20$

$-16 + 24 - 4z = 20$

$z = -3$

Solution is (-2, 6, -3)

$x - 3y + 3z = -4$

$2x + 3y - z = 15$

$4x - 3y - z = 19$

1) Pair equations to eliminate 1 variable

$$\begin{array}{r} x - 3y + 3z = -4 \\ 2x + 3y - z = 15 \\ \hline 3x + 2z = 11 \end{array} \qquad \begin{array}{r} 2x + 3y - z = 15 \\ 4x - 3y - z = 19 \\ \hline 6x - 2z = 34 \end{array}$$

2) Solve new system

$$\begin{array}{r} 3x + 2z = 11 \\ 6x - 2z = 34 \\ \hline 9x = 45 \\ x = 5 \end{array}$$

$3x + 2z = 11$

$15 + 2z = 11$

$2z = -4$

$z = -2$

$2x + 3y - z = 15$

$2(5) + 3y - (-2) = 15$

$y = 1$

Solution is (5,1, -2)

$$Solve\begin{cases} y = x^2 - x - 6 \\ y = 2x - 2 \end{cases}$$

Substitute from the linear equation into the quadratic equation and solve.

$y = x^2 - x - 6$

$2x - 2 = x^2 - x - 6$

$2x = x^2 - x - 4$

$0 = x^2 - 3x - 4$

$0 = (x - 4)(x + 1)$

$x - 4 = 0 \qquad x + 1 = 0$

$x = 4 \qquad x = -1$

Find the y- each valu

y

P

y =

P

There are 2 "possible" solutions for the system: (4,6) and (-1,-4) Check each in both equations.

$y = x^2 - x - 6$

$6 = (4)^2 - 4 - 6 = 6$ checks

$y = 2x - 2$

$6 = 2(4) - 2 = 6$ checks

$y = x^2 - x - 6$

$-4 = (-1)^2 - (-1) - 6 = -4$ checks

$y = 2x - 2$

$-4 = 2(-1) - 2 = -4$ checks

Answer:

{(4, 6), (-1, -4)}

$$\begin{cases} x^2 + y^2 = 10 \\ y = x + 2 \end{cases}$$

$x^2 + (x + 2)^2 = 10$

$x^2 + x^2 + 4x + 4 = 10$

$2x^2 + 4x - 6 = 0$

$2(x^2 + 2x - 3) = 0$

$2(x + 3)(x - 1) = 0$

$x = -3 \qquad x = 1$

$(-3, -1) \qquad (-1, 3)$