NEW OUTCOME: Analyze and make predictions about arithmetic and geometric sequences and series.

In the world around us. There are situations that involve adding or subtracting the same number repeatedly, resulting in a sequential pattern.

An ***arithmetic sequence*** is a sequence whose successive terms differ by the same number, called the ***common difference***. Using subscript notation, the general rule for an arithmetic sequence, a_1 is the first term, d is the common difference. Using function notation, the general rule is written as $a(n) = a(1) + (n - 1)d$.

PART I: Determining a Rule for Arithmetic Sequences

Some plastic chairs are designed to be stackable. After the first chair in the stack, each additional chair adds the same amount to the overall height of the stack. How can you model the height of a stack of n chairs?

The table below shows the total height of the stack when there are 1, 2, 3, and 4 chairs in the stack.

Number of chairs, n	Height of stack (in.)
1	28
2	31
3	34
4	37

(a) Look at the table. Do you notice any patterns in the stack height values? What is the common difference?

(b) Use the information from above and the general rule $a(n)$ for arithmetic sequences to create a rule for this scenario.

(c) What are the domain and range of $a(n)$?

(d) If there are 20 chairs stacked, use your rule $a(n)$ to calculate the expected height of the stacked chairs.

PART II: Determining a Term in an Arithmetic Sequence

If you know that a sequence is arithmetic and you can determine both its first term and the common difference, you can determine the value of any term in the sequence.

1. The table of values at the right gives the first four terms of an arithmetic sequence. Below we will go through the process in determining what is the 11$^{\text{th}}$ term of the sequence?

n	$a(n)$
1	-6
2	-1
3	4
4	9

To write a rule for the sequence, you need to know the first term and the common difference.

(a) What reasoning can you use to justify that the first term of the sequence is -6?

(b) How do you determine the common difference of this sequence? How can you verify this difference is correct?

(c) Use the first term, a_1, and the common difference, *d*, create a rule for this arithmetic sequence.

$$a(n) = \_\_\_\_\_ + (\_\_\_\_\_ - 1)\_\_\_\_\_$$

(d) Use the rule to determine the 11th term in this sequence.

2. An arithmetic sequence has a common difference of -10 and its 7$^{\text{th}}$ term is 140. Write a rule for the sequence. What is the 19$^{\text{th}}$ term?

The common difference is provided, but we need to determine the 1st term (a_1). We can substitute this information in and solve for the first term as shown below.

$$a(n) = a(1) + (n - 1)d$$

With $n = 7$, $d = -10$ and $a(7) = 140$

$$140 = a(1) + (7 - 1)(-10)$$

$$140 = a(1) - 60$$

$$200 = a(1)$$

We can now create a rule to solve for the 19$^{\text{th}}$ term $\rightarrow$ $a(19) = 200 + (19 - 1)(-10) = 20$

3. The 12^{th} term of an arithmetic sequence is 33 and the 16^{th} term is 41. What is a rule for the sequence, written in function notation.

(a) We can use the general rule with the given information to find two expressions for the first term, *a(1).* What can be done next in order to solve for a(1)?

$$33 = a(1) + (12 - 1)d$$

$$41 = a(1) + (16 - 1)d$$

(b) How can you use the equation 33 = a(1) + (12 - 1)(2) to finish the work needed to write the general rule for the arithmetic sequence?

(c) Write the general rule for the function.

4. Consider an arithmetic sequence {2, 5, 8, 11, 14 … }. What is the sum of the first 25 terms?
[Hint: Use the formula from Monday's lesson for the sum of a sequence]