TASK: If you remember back to Algebra 2, we had different methods to solve exponential and logarithmic equations. Below is a summary of those different types of methods and when is appropriate for each. Read through each and practice. You may want to pull your Algebra 2 notes and use it as a study tool as well.

A. Using Like Bases to Solve Exponential Equations

The first technique involves two functions with like bases. Recall that the one-to-one property of exponential functions tells us that, for any real numbers b, S, and T, where $b > 0$, $b \neq 1$, $b^S = b^T$ if and only if $S = T$. Given an exponential equation with the form $b^S = b^T$, where s and t are algebraic expressions with an unknown, solve for the unknown:

1. Use the rules of exponents to simplify, if necessary, so that the resulting equation has the form $b^S=b^T$ (with the same bases).
2. Use the one-to-one property to set the exponents equal.
3. Solve the resulting equation, $S=T$, for the unknown.

Solve $3^{4x-7} = \frac{3^{2x}}{3}$.

Solution

$3^{4x-7} = \frac{3^{2x}}{3}$

$3^{4x-7} = \frac{3^{2x}}{3^1}$ Rewrite 3 as 3^1

$3^{4x-7} = 3^{2x-1}$ Use the division property of exponents

$4x - 7 = 2x - 1$ Apply the one-to-one property of exponents

$2x = 6$ Subtract 2x and add 7 to both sides

$x = 3$ Divide by 3

***Sometimes you may first have to make the bases the same**

- Example: We can express 27 as 3^3, an equivalent form (but we didn't actually change the value)

Solve $8^{x+2} = 16^{x+1}$.

Solution

$8^{x+2} = 16^{x+1}$

$(2^3)^{x+2} = (2^4)^{x+1}$ Write 8 and 16 as powers of 2

$2^{3x+6} = 2^{4x+4}$ To take a power of a power, multiply exponents

$3x + 6 = 4x + 4$ Use the one-to-one property to set the exponents equal

$x = 2$ Solve for x

B. Solving Exponential Equations Using Logarithms

Sometimes the terms of an exponential equation cannot be rewritten with a common base. In these cases, we solve by taking the logarithm of each side. Recall, since log (a) = log (b) is equivalent to $a = b$, we may apply logarithms with the same base on both sides of an exponential equation. Given an exponential equation in which a common base cannot be found, solve for the unknown:

1. Apply the logarithm of both sides of the equation.
2. Choice 1: If one of the terms in the equation has base 10, use the common logarithm.
   Choice 2: If none of the terms in the equation has base 10, use the natural logarithm.
3. Use the rules of logarithms to solve for the unknown.

Solve $5^{x+2} = 4^x$.

Solution

$5^{x+2} = 4^x$	There is no easy way to get the powers to have the same base
$\ln 5^{x+2} = \ln 4^x$	Take ln of both sides
$(x+2)\ln 5 = x \ln 4$	Use laws of logs
$x \ln 5 + 2\ln 5 = x \ln 4$	Use the distributive law
$x \ln 5 - x \ln 4 = -2 \ln 5$	Get terms containing x on one side, terms without x on the other
$x(\ln 5 - \ln 4) = -2 \ln 5$	On the left hand side, factor out an x
$x \ln\left(\frac{5}{4}\right) = \ln\left(\frac{1}{25}\right)$	Use the laws of logs
$x = \dfrac{\ln\left(\frac{1}{25}\right)}{\ln\left(\frac{5}{4}\right)}$	Divide by the coefficient of x

PRACTICE:

Solve each equation

1. $5^x = \sqrt{5}$
2. $5^{2x} = 25^{3x+2}$
3. $4^{2a} = \frac{1}{16}$
4. $256 = 4^{x-5}$
5. $8 \cdot 2^{2p} = 1$
6. $3^{-2m} \cdot 3^{3m} = 3^{-m-3}$
7. $2^x = 3^{x+1}$
8. $\dfrac{25^x}{\left(\frac{1}{5}\right)^{-x-3}} = 125^{-2x}$
9. $10^{2t-3} = 7$
10. $\frac{1}{2}(10^{x-1})^x + 3 = 53$