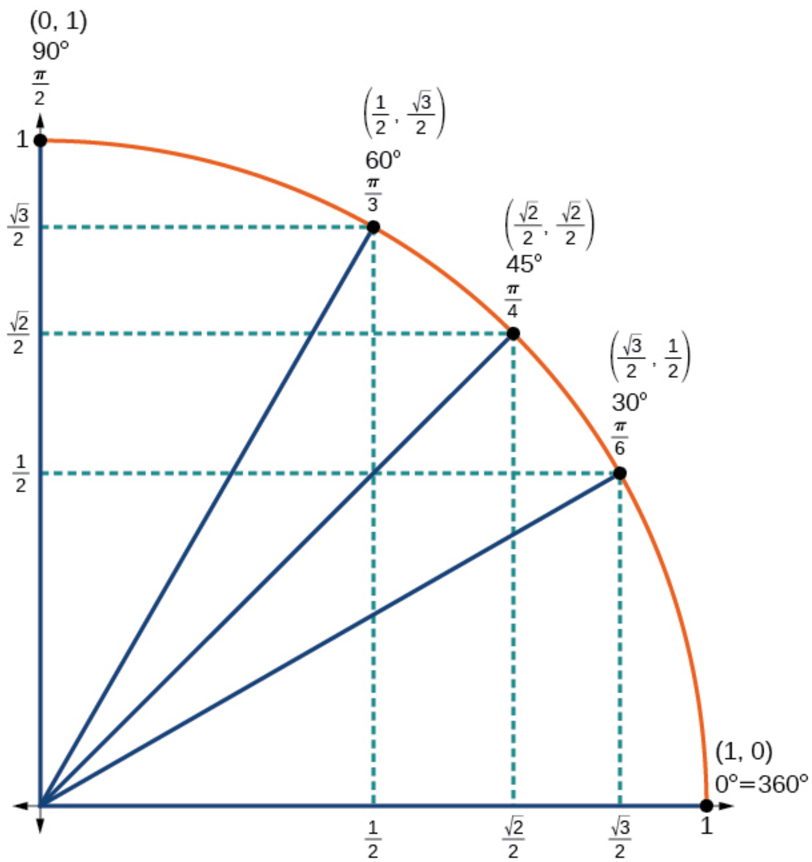
Using Sum & Difference Formulas:

Finding the exact value of the sine, cosine, or tangent of an angle is often easier if we can rewrite the given angle in terms of two angles that have known trigonometric values. We can use the special angles on the unit circle we memorized in Algebra 2, shown below.



Do you have these memorized? Remember how to determine the reference angles if you don't get one of these angles? Here is a helpful way Katherine has them memorized and steps reviewed.

Helpful steps:

1. Convert angles from radians to degrees.
   - *Sometimes you may need to determine the reference angle if the degree amount is not 30, 45, or 60.*
2. Write the appropriate formula for the trig function.
3. Substitute the values of the given angles into the formula
   - *Sometimes you may have to use trig ratios or the pythagorean identity to solve for a value from the given information.*
4. Simplify.
5. If you have time to check your answer in your calculator, make sure you are aware of the need to be in radian or degree mode.

Sum & Difference of Cosine:

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

<u>Model Example:</u> Find the exact value of $cos\left(\frac{5\pi}{4} - \frac{\pi}{6}\right)$

Begin by converting and writing the formula for the cosine of the difference of two angles. Then substitute the given values.

$cos\,(\alpha + \beta) = cos\,\alpha\, cos\,\beta - sin\,\alpha\, sin\,\beta$

$cos\,(225 + 30) = cos\,225\, cos\,30 - sin\,225\, sin\,30$

225 is not a special angle, but the reference angle for 225 is 45 (and in quadrant III Cos is negative)

$= (-\,cos\,45)\, cos\,30 - sin\,45\, sin\,30$

$= \left(\frac{-\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{-\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{-\sqrt{6} - \sqrt{2}}{4}$

<u>You try an example:</u> Find the exact value of $cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

Occasionally, you may need to create the sum or difference using the special angles you have memorized.

<u>Model Example:</u> Find the exact value of $cos\,(75\,^\circ)$

Begin replacing the degree amount with a sum or difference composed of the special angles 30, 45, 60, or the quadrantal angles 90, 180, 270, 360, etc.

$cos\,(\alpha + \beta) = cos\,\alpha\, cos\,\beta - sin\,\alpha\, sin\,\beta$

$cos\,(75) = cos\,(45 + 30) = cos\,45\, cos\,30 - sin\,45\, sin\,30$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$

<u>You try an example:</u> Find the exact value of $cos\,(105\,^\circ)$

Sum & Difference of Sine:

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

**Same steps as for Cosine*

<u>You try an example:</u> Find the exact value of $sin\,(135^{\circ} - 120^{\circ})$

Sum & Difference of Tangent:

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

**Same steps as Sine & Cosine, but remember to determine* $tangent = \frac{Sine}{Cosine}$

<u>You try an example:</u> Find the exact value of $tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$