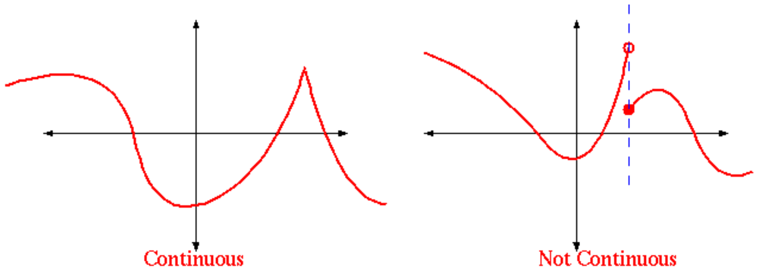
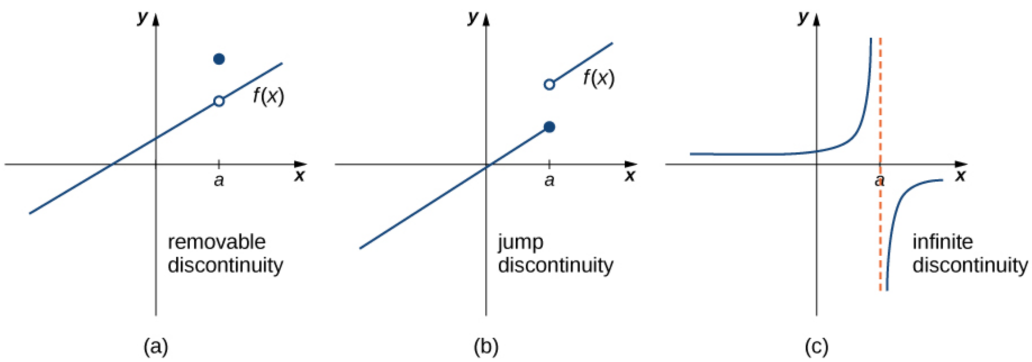
What does it mean for a function to be continuous?

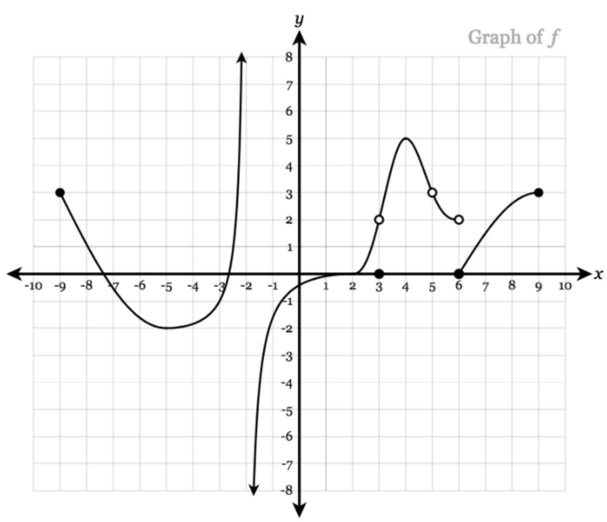
A function f is said to be ***continuous*** at a point a if and only if the limit of f as x approaches a is the value of $f(a)$ of f at a. That is: $\lim_{x \to a} f(x) = f(a)$



Types of Discontinuities:



Model Example:



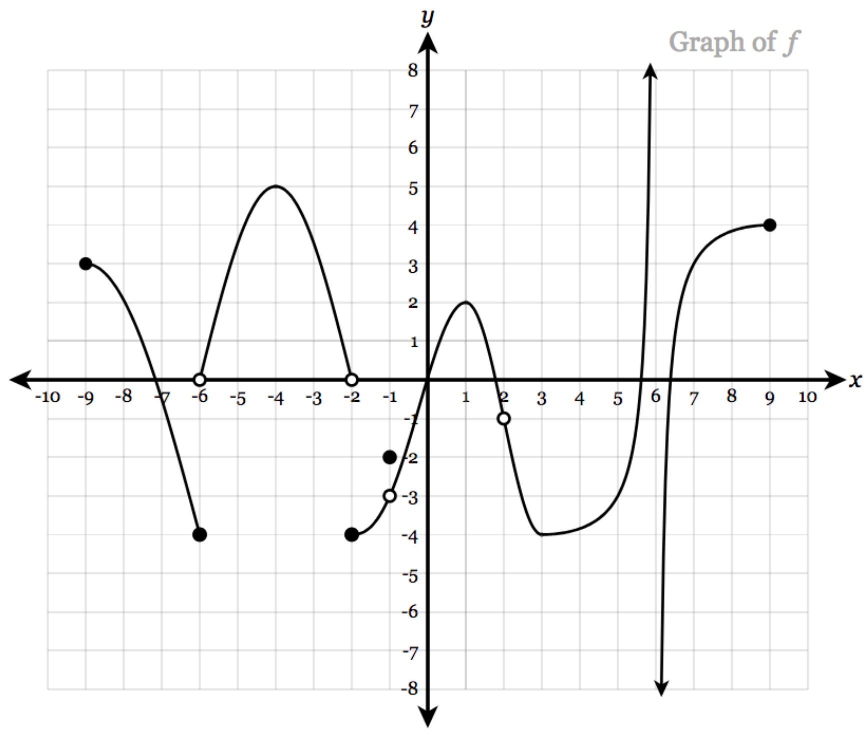
At x = -2 there is *infinite* discontinuity.

At x = 3 there is *removable* discontinuity.

At x = 5 there is *removable* discontinuity.

At x = 6 there is *jump* discontinuity.

You Try: State all the values where f(x) has a discontinuity and identify the type of discontinuity.



Determining Continuity Algebraically: We can approach this question with limits and find if the left and right of -2 are approaching the same value.

Determine whether the function $f(x)$ is continuous at $x = -2$.

$$f(x) = \begin{cases} 18 - 5x^2, & x > -2 \\ 2 + 2x, & x \leq -2 \end{cases}$$

$$\lim_{x \to -2-} 2 + 2(-2) = -2$$

$$\lim_{x \to -2+} 18 - 5(-2)^2 = -2$$

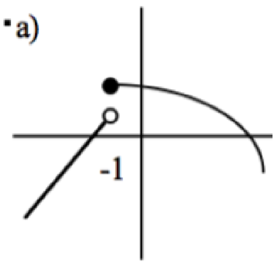
Therefore, this function is continuous at -2.

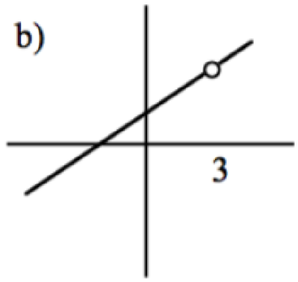
[This video explains continuity with rational functions (and is a nice review)]

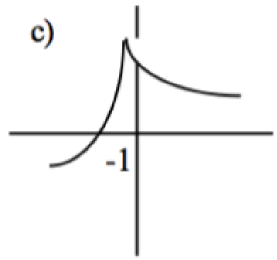
Practice:

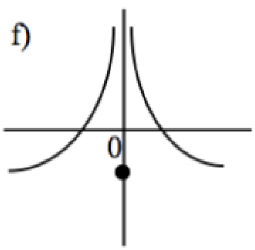
VI. Determine Continuity and Identify Points of Discontinuity (If any exist)

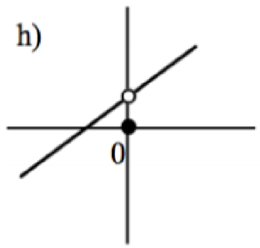
1.

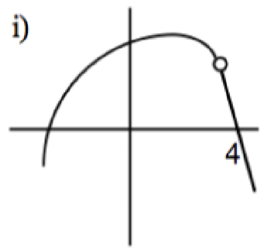












2. $f(x) = x^3 - 2x^2$

3. $f(x) = \dfrac{x-4}{x^2 + x - 20}$

4. $f(x) = \tan(x)$

5. $f(x) = \sin(x)$

6. $f(x) = \begin{cases} 8 - x^2, x < 2 \\ 6 - x, x \geq 2 \end{cases}$

7. $f(x) = \begin{cases} 4 - x^2, x < 1 \\ 1 + x, x \geq 1 \end{cases}$