

How do we create representations using limits?

Here we are going to see how to sketch a graph of a function with limits.

Model Example 1: Sketch the graph of a function f that satisfies the given values :

- $f(0)$ is undefined
- $\lim_{x \rightarrow 0} f(x) = 4$
- $f(2) = 6$
- $\lim_{x \rightarrow 2} f(x) = 3$

SOLUTION:

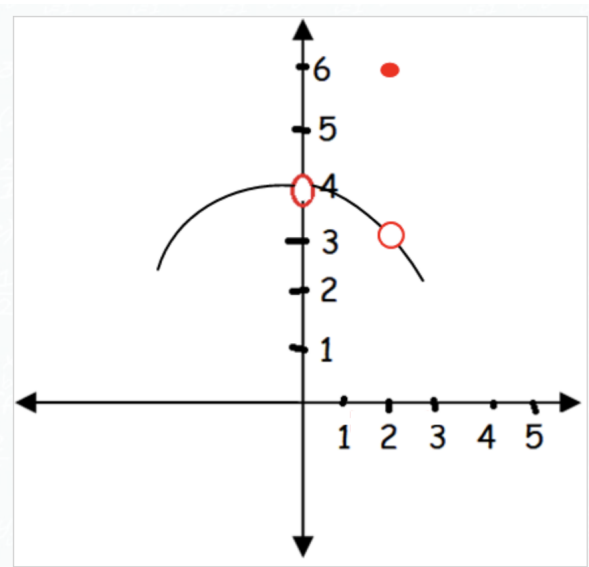
From the given question,

\Rightarrow we understand that the function is undefined when $x = 0$.

\Rightarrow when the value of x approaches 0 from left hand side and right hand side, limit value will approaches to 4.

\Rightarrow when $x = 2$, the value of $f(x)$ will be 6.

\Rightarrow when the value of x approaches 2 from left hand side and right hand side, limit value will approaches to 3.



Model Example 2:

Graph the function

$$f(x) = \begin{cases} 2x - 4 & \text{if } x \neq 3 \\ -5 & \text{if } x = 3. \end{cases}$$

Use the graph to find $\lim_{x \rightarrow 3} f(x)$.

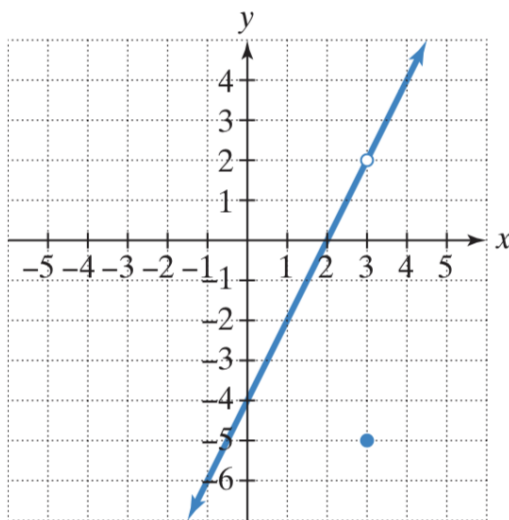
SOLUTION:

This piecewise function is defined by two equations. Graph the piece defined by the linear function, $f(x) = 2x - 4$, using the y-intercept, - 4, and the slope, 2. Because $x = 3$ is not included, show an open dot on the line corresponding to $x = 3$. This open dot, with coordinates $(3, 2)$, is shown in the graph below.

Now we complete the graph using $f(x) = - 5$ if $x = 3$. This part of the function is graphed as the point $(3, -5)$, shown as a closed blue dot in the graph..

To find $\lim_{x \rightarrow 3} f(x)$ examine the graph of f near $x = 3$. As x gets closer to 3, the values of $f(x)$ get closer to the y-coordinate of the point shown by the open dot. The y-coordinate of this point is 2. We conclude from the graph that

$$\lim_{x \rightarrow 3} f(x) = 2$$



PRACTICE:

1. Graph the function

$$f(x) = \begin{cases} 3x - 2 & \text{if } x \neq 2 \\ 1 & \text{if } x = 2. \end{cases}$$

Use the graph to find $\lim_{x \rightarrow 2} f(x)$.

2. Sketch a graph of a function g that satisfies all of the following conditions.

- $g(3) = -1$
- $\lim_{x \rightarrow 3} g(x) = 4$
- $\lim_{x \rightarrow -2^+} g(x) = 1$
- g is increasing on $-2 < x < 3$
- $\lim_{x \rightarrow -2^-} g(x) > \lim_{x \rightarrow -2^+} g(x)$

